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A METHOD OF ESTIMATING THE DOMAIN OF CONTROLLABILITY IN NON-LINEAR SYSTEMS*

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An estimate of the domain of controllability is given for a class of non-linear control systems. The domain is the region of phase space from every point of which the system can be taken to any other point by means of an admissible control.

1. Consider the differential equations of perturbed motion of a control system (written in vector notation)

$$x^{\cdot} = \Phi(x, u), \quad \Phi \in C^1(R^n \times Q_0), \quad Q_0 \subseteq R^m \tag{1.1}$$

Here u is the control vector acting from the bounding set $Q \subseteq Q_0$ for which the vector $u = 0$ is an internal point and R^n, R^m are real n- and m-dimensional spaces respectively $(m \leqslant n)$.

Following /1/ we shall define the domain of zero controllability as the set of initial points $x_0 \in R^n$ from which system (1.1) can be brought to the point $x = 0$ by means of the bounded measurable controls $u(t) \in Q$ defined in some finite time interval.

We know /1/ that if a positive definite Lyapunov function $V(x)$ and an m-dimensional vector function $u_R(x)$ exists for the control system in R^n of class C^1, such that

$$\lim_{|x| \to \infty} V(x) = \infty; \quad \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \Phi_i(x, u_R(x)) < 0, \quad x \neq 0$$

$\Phi(0, 0) = 0$, ($|\cdot|$ is the length of the vector) and the condition of local controllability

$$\operatorname{rank}\,[B, AB, \ldots, A^{n-1}B] = n, \quad 0 \in Q, \quad A = \Phi_x(0, 0) \tag{1.2}$$
$$B = \Phi_u(0, 0)$$

holds, then the domain of zero controllability for system (1.1) is identical with R^n.

Let us consider the case in which a constructive estimate can be given for the domain of zero controllability.

Let a Lyapunov function $V(x)$ be known for system (1.1) without control, positive definite in the region $\Omega_0 \subset R^n$, $0 \in \Omega_0$, the time derivative of which

$$V_0^{\cdot} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \Phi_i(x, 0),$$

by virtue of (1.1) (with $u \equiv 0$), is a non-positive function in Ω_0.

We take a function $U(u) \in C^1(Q_0)$, positive definite in $u \in Q_0$, e.g. a positive definite quadratic form. We write the expression

$$F_\Sigma(x, u) \equiv \sum_{i=1}^{m} \frac{\partial V}{\partial x_i} [\Phi_i(x, u) - \Phi_i(x, 0)] + U(u) \tag{1.3}$$

are separate it into m components.

$$F_\Sigma(x, u) = \sum_{k=1}^{m} F_k(x, u)$$

$$F_k(x, u) \equiv \sum_{j=1}^{n_k} \frac{\partial V}{\partial x_j} [\Phi_j(x, u) - \Phi_j(x, 0)] + U_k(u)$$

$$U_k(u) \geqslant 0; \quad \sum_{k=1}^{m} U_k(u) = U(u); \quad k = 1, \ldots, m; \quad n_1 + \ldots + n_m = n$$

Let us separate from the functions $F_k(x, u)$ the cofactors of the form $F_k^{**}(u)$, provided that they exist, i.e. let us write $F_k(x, u)$ in the form $F_k(x, u) = F_k^*(x, u) F_k^{**}(u)$ and consider the system of equations

$$F_k^*(x, u) = 0, \quad k = 1, \ldots, m \tag{1.4}$$

The vector function $F^*(x, u)$ is defined and continuously differentiable in the region

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$R^n \times Q$, and $F^*(0, 0) = 0$.

We shall assume that the functional determinant $|\partial F_k^*(x, u)/\partial u_j|$ is non-zero at every point (x, u) of the open set $S \subset R^n \times Q_0$ containing the point $x = 0$, $u = 0$.

Let R_x, R_u be two positive numbers such that when $|x| \leqslant R_x$, $|u| \leqslant R_u$, the point (x, u) belongs to the open set S and the following inequality holds for it:

$$\left|\frac{\partial}{\partial u_j} g_k(x, u)\right| \leqslant \frac{a}{n^2}, \quad k, j = 1, \ldots, m \tag{1.5}$$

Here a is any number from the interval $(0, 1)$, $g_k(x, u)$ is the coordinate of the vector function $g(x, u)$

$$g(x, u) = u - B^{-1}F^*(x, u) \tag{1.6}$$

and B is a matrix of the coefficients $b_{kj} = \partial F_k^*(0, 0)/\partial u_j$.

Then it follows from the theorem on the implicit function /2/ that a continuous and unique solution $u = u_c(x)$, $u_c(0) = 0$ of the equation $F^*(x, u) = 0$ exists in the open set Ω_1 defined by the inequality $|x| < R_{0x} \leqslant R_x$ and

$$|g(x, 0) - g(0, 0)| < (1 - a) R_u \tag{1.7}$$

(Under the assumption used a number R_{0x} must exist and can be computed.)

Some arbitrariness in choosing the function $U(u)$ and in splitting the expression $F_\Sigma(x, u)$ into its terms can be utilized for determining the control $u_c(x)$ in the easiest possible way.

For example, if the functions $\Phi_i(x, u)$ can be written in the form

$$\Phi_i(x, u) = \Phi_i(x) + \sum_{j=1}^{m} \Phi_{ij}(x) u_j + \sum_{k, j=1}^{m} \Phi_{ikj}(x) u_i u_j \tag{1.8}$$

then, assuming that

$$U(u) = \sum_{k, j=1}^{m} \alpha_{kj} u_k u_j$$

we obtain

$$F_\Sigma(x, u) \equiv \sum_{j=1}^{m}\Big(\sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \Phi_{ij}(x)\Big) u_j + \sum_{k, j=1}^{m}\Big(\sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \Phi_{ikj}(x) + \alpha_{kj}\Big) u_k u_j$$

Therefore terms of the type $F_k^*(x, u)$ and equations (1.4) are obtained here quite naturally

$$\sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \Phi_{ij}(x) + \sum_{k=1}^{m}\Big(\sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \Phi_{ikj}(x) + \alpha_{kj}\Big) u_k = 0$$

Thus in the case in question, the problem of determining the control reduces to that of solving a system of linear equations.

From the construction of the control $u = u_c(x)$ it is clear that it will stabilize the unperturbed motion $x = 0$ of the system (1.1), provided that the manifold $M \subset R^n$ defined by the expression $V_0^{\cdot}(x) - U(u_c(x)) = 0$ does not, according to the Barbashin-Krasovskii theorem, contain complete trajectories of system (1.1).

Let the limiting set Q depend on x and be defined by the expression

$$Q(x) = \{|u_j| \leqslant u_{0j}(x) \subset C(R^n),\ 0 < u^* = \text{const} \leqslant u_{0j}(x) \leqslant u^{**} = \text{const},\ j = 1, \ldots, m\} \tag{1.9}$$

If $Q_1 \setminus Q \neq 0$, then we define the controls thus

$$u_j = u_j^\circ(x) = \begin{cases} u_{0j}(x) \operatorname{sign} u_{cj}(x), & |u_{cj}(x)| \geqslant u_{0j}(x) \\ u_{cj}(x), & |u_{cj}(x)| < u_{0j}(x);\ ((u_{c1}, \ldots, u_{cm}) = u_c) \end{cases} \tag{1.10}$$

Since the controls $u_j^\circ(x)$ are continuous, it follows that the derivative $V^{\cdot}$, by virtue of the system (1.1), will be defined in Ω_1 when $u_j = u_j^\circ(x)$. Let every one of the sums

$$\Sigma_k = \sum_{j=1}^{n_k} \frac{\partial V}{\partial x_j} \Phi_j(x, u), \quad k = 1, \ldots, m$$

be able to change its sign for fixed $x \in \Omega_1$ only when all controls entering Σ_k change their signs simultaneously. Then the derivative $V^{\cdot}$, by virtue of (1.1), will vanish when $u_j = u_j^\circ(x)$ only when $x \in M$.

Consequently, system (1.1) can be stabilised by bounded controls belonging to the set $Q(x)$, i.e. the region Ω_1 lies in the domain of zero controllability of (1.1). Clearly, in the case

(1.8) $\Omega_1 = \Omega_0$.

Note that the use of the well-known Lyapunov function to construct the optimal stabilizing controls for system (1.1) was suggested earlier in /3, 4/ for the case when the functions $\Phi_i(x, u)$ are polynomials of first and second degree with respect to the controls.

In some cases the proof of stabilizability of the unperturbed motions of system (1.1) in the region Ω_0 can be simplified by introducing a new control vector v connected with the initial control vector u by the relation $v = v(u)$, single-valued and continuous in the neighbourhoods Q_u, Q_v of the points $u = 0$, $v = 0$, under which the right-hand side of the system becomes simpler, e.g. becomes linear with respect to the new controls, i.e.

$$\Phi_i(x, u(v)) = \Phi_{0i}(x) + \sum_{j=1}^{m} \Phi_{ij}(x) v_j, \quad i = 1, \ldots, n \tag{1.11}$$

We shall illustrate this by an example. The problem of optimal stabilization of the stationary motion of a satellite about its centre of mass was studied in /5/. The satellite was situated at the triangular libration point of the system of two bodies, and the stabilization was carried out by varying the moments of inertia of the satellite. The right-hand side of the equations of perturbed motion of such a control system has the form (1.11).

The stationary motion of the satellite, stable in the region of librational motion Ω_0 is stabiliized by the controls v_j $(j = 1, 2, 3)$ with or without constraints of the type (1.9). The controls v_j in this problem are functions of the displacements u_j of the centres of mass of the displaced massive bars, and have the form

$$v_1 = \frac{1}{w_1} - \frac{1}{B_0}, \quad v_2 = \frac{w_2}{w_4} - \frac{A_0 - B_0}{A_0 + C_0}, \quad v_3 = \frac{w_3}{w_4} - \frac{A_0 - C_0}{A_0 + C_0} \tag{1.12}$$

where

$$w_1 = B_0 + \lambda_1 u_1 + \lambda_3 u_3 + m_1 u_1^2 + m_3 u_3^2$$
$$w_2 = A_0 - B_0 + \lambda_2 u_2 - \lambda_3 u_3 + m_2 u_2^2 - m_3 u_3^2$$
$$w_3 = A_0 - C_0 - \lambda_1 u_1 + \lambda_3 u_3 - m_1 u_1^2 + m_3 u_3^2$$
$$w_4 = A_0 + C_0 + \lambda_1 u_1 + 2\lambda_2 u_2 + \lambda_3 u_3 + m_1 u_1^2 + 2m_2 u_2^2 + m_3 u_3^2$$

and A_0, B_0, C_0. $\lambda_1, \lambda_2, \lambda_3, m_1, m_2, m_3$ are constant positive parameters of the problem $(B_0 > C_0 > A_0)$.

To estimate the regions Q_u and Q_v we shall use the expressions (1.5), (1.6), (1.7) in which the functions $F^*(x, u)$, $g(x, u)$ are replaced by the corresponding functions $F(v, u)$, $g(v, u)$ with coordinates

$$F_j(v, u) = \zeta_j - v_j, \quad g_j(v, u) = u_j - \langle B_j, F(v, u)\rangle$$
$$g_j(0, 0) = 0, \quad g_j(v, 0) = \sum_{k=1}^{3} B_{jk} v_k, \quad j = 1, 2, 3$$

Here $B_j = (B_{j1}, B_{j2}, B_{j3})$. B_{jk} are elements of the matrix B^{-1}, $\langle \cdot, \cdot \rangle$ is the scalar product of the vectors and ζ_j are the corresponding right-hand sides of expressions (1.12).

Since the derivatives $\partial g_k(v, u)/\partial u_j$ are smooth, depend on the function u only and vanish when $u = 0$, after passing from the inequalities of the type (1.5) to equalities (at fixed a), we obtain equations of the surfaces bounding the region Q_u. Apart from these surfaces, the region Q_u is also bounded by the surface representing the boundaries of the domain of definition of $F_j(v, u)$ and described by the equations $w_1(u) = 0$, $w_4(u) = 0$. Therefore the quantity R_u represents here the minimum distance separating the point $u = 0$ from these surfaces. Inequalities (1.7) have the form

$$\left| \sum_{k=1}^{3} B_{jk} v_k \right| < (1 - a) R_u, \quad j = 1, 2, 3$$

Thus we find that the region Q_v is bounded by the surfaces

$$\sum_{k=1}^{3} B_{jk} v_k = \pm (1 - a) R_u, \quad j = 1, 2, 3$$

If R_{0v} is the minimum distance from these surfaces to the point $v = 0$, then satisfying the condition $u^{**} < {}^2/_3 R_{0v}$ guarantees the solution of the problem of stabilizing the stationary motion of the satellite by means of controls u_1, u_2, u_3 with constraints of the type (1.9).

Returning now to the general case, we shall assume that the estimate of the region Ω_1 for the control system (1.1) is known.

If condition (1.2) also holds for such a system, then according to Theorem 1 of /1/ its domain of zero controllability is open in R^n. Consequently, any point of the region Ω_1 can be transported by an admissible control, i.e. a measurable function $u(t) \in Q(x)$, to the origin of coordinates over a finite period of time.

Note the following properties of the control system (1.1), (1.2), (1.9). The set $Q(x)$ is closed, bounded, and by virtue of the continuity of the bounding functions $u_{0j}(x)$ semicontinuous from above with respect to the inclusion (in x), i.e. the following assertion holds:

$$(\forall x \in R^n)\ (\forall \varepsilon > 0)\ (\exists \delta = \delta(\varepsilon, x) > 0)\ (\forall x',\ |x - x'| < \delta) : Q(x') \subseteq Q_\varepsilon(x)$$

Here $Q_\varepsilon(x) - \varepsilon$ denotes the neighbourhood of the set $Q(x)$.

When the vector u traverses the set $Q(x)$, the vector function $\Phi(x, u)$ traverses the set $R(x)$ which is also semicontinuous from above with respect to the inclusion.

Moreover, in the case of a linear dependence of $\Phi(x, u)$ on u the set $R(x)$ is convex (for any $x \in R^n$).

In addition to the properties listed above for the system (1.1), let the set $R(x)$ be convex and let the following inequality hold /6/:

$$\langle x, \Phi(x, u)\rangle \leqslant c\,(|x|^2 + 1), \quad c = \text{const}, \quad u \in Q(x)$$

Then the condition of Theorem 1 of /6/ will hold for every point $x \in R^n$. The theorem implies that for every point of the region Ω_1 an admissible control exists which transfers this point to the origin of coordinates in the shortest possible time.

2. Let us now consider the problem of estimating the region of zero controllability for an autonomous control system written in the form of the Lagrange equations

$$\frac{d}{dt}\frac{\partial T}{\partial q_i^{\cdot}} - \frac{\partial T}{\partial q_i} = -\frac{\partial \Pi}{\partial q_i} + \sum_{j=1}^{m} F_{ij}(q)\, u_j \tag{2.1}$$

without assuming that the stationary motion $q_i = 0,\ q_i^{\cdot} = 0,\ (i = 1, \ldots, n)$ is stable when $u_j \equiv 0\ (j = 1, \ldots m)$.

Let the potential energy of the system (2.1) have a maximum equal to zero $\Pi(0) = 0$, when $q = 0$, with bounding functions independent of the generalized velocities, i.e. when we have in conditions (1.9) $u_{0j} = u_{0j}(q)$.

We shall write the controls in the form

$$u_j = v_j + w_j, \quad |v_j| \leqslant \mu u_{0j}(q), \quad \mu \in (0, 1) \tag{2.2}$$
$$\sum_{j=1}^{m} F_{ij}(q)\, w_j = (1 + \mu)\frac{\partial \Pi}{\partial q_i}, \quad i = 1, \ldots, n$$

and assume that equations (2.2) are compatible and have the solutions $w_j = w_j^*(q),\ w_j^*(0) = 0$ in the region G_F containing the point $q = 0$ of the configurational space G. We define the surfaces γ_j^+, γ_j^- in the region G_F by the equations $\pm(1 - \mu)\, u_{0j}(q) = w_j^*(q)$ respectively. (If the latter equations have no solutions for some values of j, there are no corresponding surfaces).

The region G_μ containing the point $q = 0$ and bounded by the surfaces γ_j^+, γ_j^- and the boundary of G_F, is obviously non-empty for sufficiently small μ. A value $\mu = \mu^*$ exists on the bounded set $\mu \in (0, 1)$, for which the region G_μ will contain the closed surface of constant level of the function $(-\mu^*\Pi)$ defined by the equation $-\mu^*\Pi = \Pi^*,\ \Pi^* = \text{const} > 0$, farthest from the origin of coordinates.

Let us write the equations corresponding to the control system (2.1), after passing to the new controls

$$\frac{d}{dt}\frac{\partial T}{\partial q_i^{\cdot}} - \frac{\partial T}{\partial q_i} = \mu\frac{\partial \Pi}{\partial q_i} + \sum_{j=1}^{m} F_{ij}(q)\, v_j \tag{2.3}$$

We find that for any initial perturbations belonging to the region Ω^* of phase space R^{2n} bounded by the surface of integral manifold of the system (2.3), when $u \equiv 0$ and

$$H(q, q^{\cdot}) \equiv T(q, q^{\cdot}) - \mu\Pi(q) = \Pi^*$$

the unperturbed motion $q = 0,\ p = 0$ is stable and can be stabilized by a continuous bounded control. Consequently, under the additional assumptions given in Sect.1, the region Ω^* can serve as an estimate for the region of zero controllability.

Note that the addition to the system (2.1) of the non-potential forces which vanish at the origin of coordinates, does not cause any difficulties in principle in determining the region Ω^*.

The homogeneous walk of a plane non-linear model of a walking device along a horizontal plane, is described /7/ by a system of five differential equations of the form (2.3). The problem of bringing the device to the upper unstable equilibrium position in the least time may be of interest. Since the dimension of the control vector is equal here to the number of equations, it follows that the system of the type (2.2) is consistent.

In the linear approximation the equations of motion of a walking device near its upper position of unstable equilibrium have the form

$$\sum_{j=1}^{5} a_{ij} q_j^{\cdot\cdot} = \sum_{j=1}^{5} b_{ij} q_j + u_i, \quad b_{ij} = b_{ji}, \quad i, j = 1, \ldots, 5 \tag{2.4}$$

Since the dimensions of the control vector is equal to the number of equations, the conditions of controllability of system (2.4) are satisfied. (In the case of a scalar control the conditions of controllability of the system of the form (2.4) were obtained in /8/). Other conditions for the region Ω^* (Sect.1) to exist are also satisfied.

3. Let us now consider the problem of estimating the region of total controllability of system (1.1), i.e. of a set such that a phase point can be taken from any point of the set, by means of an admissible control, to any point of this set in a finite time.

Let $D(t)$ be a set of points $x \in R^n$ into which the phase point can be taken from its initial position $x = 0$ at $t = 0$, using an admissible control, in a time $t > 0$, i.e. let $D(t)$ be the region of zero attainability /1/.

It was shown in /1/ that, for the linear control systems, under the condition (1.2), the set $D(t)$ is compact, convex and depends continuously on t. Using the method Theorem 1 of /1/, we can establish that when the condition of local controllability of system (1.1) holds the set $D(t)$ has an open neighbourhood E containing the point $x = 0$. In addition, the set $D(t)$ is connected and $D(t_1) \subseteq D(t_2)$, if $t_1 \leqslant t_2$. Estimates for such sets are obtained in /9/.

Let D_∞ be the union of all sets $D(t)$ for $t < \infty$. We shall show that for a Hamiltonian control system

$$q^\cdot = \partial H(q, p)/\partial p, \ p^\cdot = -\partial H(q, p)/\partial q + F(q, p)\, u, \ (q, p) \in R^{2n} \tag{3.1}$$

satisfying the demands of Sect.1, the set D_∞ contains the region Ω_0 (Sect.1).

If system (3.1) can be stabilized by a continuous admissible control $u = u_c$ and $M \cap \Omega_0 = 0$, then the truth of the assertion is obvious.

Let $M \cap \Omega_0 \neq 0$. We shall assume that $\Gamma = \Omega_0 \setminus D_\infty \neq 0$. Let us consider a sequence of integral manifolds $I(h)$ of the system (3.1) $(u \equiv 0)$ corresponding to the energy integrals $H(q, p) = h$, $H(0, 0) = 0$ as $h \to 0, (h \geqslant 0)$ and containing the points of the set Γ. Since an open neighbourhood $E \subset D_\infty$ exists, the inequality $\inf h = h_* > 0$ must hold.

Let us choose a sequence of points $x_h \in I(h) \cap \Gamma$ converging to the point $x_* \in I(h_*)$ as $h \to h_*$. Since replacing the stabilizing control $u = u_c$ by the control $u = -u_c$ causes the phase points, except for the points belonging to the manifold M, to move towards the boundary of the region Ω_0 intersecting the surface $I(h_*)$ at the angles different from zero, it follows that $x_* \in M$.

The point x_* cannot belong to the set Γ, since when $u \equiv 0$, it must leave the set M after a finite period of time (since M contains no whole trajectories) and arrive, after a finite period of time, at the point x_H of the surface $I(h_*)$ through which the trajectory passes, intersecting the surface $I(h_*)$ without touching when $u = -u_c$. As the solutions depend on the initial conditions in a continuous manner, a neighbourhood σ of the point x_H exists in which the trajectory intersects when $u = -u_c$, the integral surfaces, without touching.

Since x_* is the tangent point of the set Γ, it follows that for any number $\varepsilon > 0$ a sufficiently small $\delta > 0$ exists for which the integral manifold $I(h_* + \delta)$ contains the point $x_\delta \in \Gamma$, $|x_H - x_\delta| < \varepsilon$. Clearly, the trajectory γ passing through the point x_δ belongs completely to the set Γ when $u \equiv 0$. Since the solutions depend continuously on the initial conditions (when $u \equiv 0$), it follows that for sufficiently small ε the trajectory must intersect the neighbourhood σ. Therefore, the phase flux of system (3.1), when $u = -u_c$, transfers some points of the set D_∞ to some points of the trajectory γ, which contradicts the initial assumption. Therefore the set Γ is empty and $\Omega_0 \subseteq D_\infty$.

(We have, by analogy, $\Omega^* \subseteq D_\infty$.) for Sect.2).

The result obtained can be used, in particular, to transfer a phase point from one stable position of equilibrium to another stable or unstable position of equilibrium, provided that the correpsonding boundaries of the regions Ω_0 have a common point at which the condition of local controllability holds. Such conditions are satisfied e.g. in the problem of the optimal reorientiation of a satellite /10/.

Let us consider one more example. We know /11/ that a gyroscopic pendulum can be stabilized in the upper unstable position of equilibrium by a single control momentum directed along the axis of rotation of the Cardan frame. It can be verified that there the region Ω_0 containing the lower stable postion of equilibrium of a gyroscopic pendulum is adjacent to the unstable upper position of equilibrium in which the condition of local controllability is satisfied. Consequently, the gyroscopic pendulum can be transferred from any point of the region Ω_0 and in particular from the lower position of equilibrium, to the upper position of equilibrium by means of an admissible control in the least possible time.

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